

Asteroseismological constraints on the coolest GW Vir variable star (PG 1159-type) PG 0122+200

A. H. Córscico^{1,2*}, M. M. Miller Bertolami^{1,2**}, L. G. Althaus^{1,2*}, G. Vauclair³, and K. Werner⁴

¹ Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque S/N, (1900) La Plata, Argentina.

² Instituto de Astrofísica La Plata, IALP, CONICET-UNLP

³ Universite Paul Sabatier, Observatoire Midi-Pyrenees, CNRS/UMR5572, 14 Av. Belin, 31400 Toulouse, France.

⁴ Institut für Astronomie und Astrophysik, Universität Tübingen, Sand 1, 72076 Tübingen, Germany.

e-mail: acorsico, althaus, mmiller@fcaglp.unlp.edu.ar; gerardv@ast.obs-mip.fr; werner@astro.uni-tuebingen.de

Received; accepted

ABSTRACT

Aims. We present an asteroseismological study on PG 0122+200, the coolest known pulsating PG1159 (GW Vir) star. Our results are based on an augmented set of the full PG1159 evolutionary models recently presented by Miller Bertolami & Althaus (2006).

Methods. We perform extensive computations of adiabatic g -mode pulsation periods on PG1159 evolutionary models with stellar masses ranging from 0.530 to $0.741 M_{\odot}$. These models take into account the complete evolution of progenitor stars, through the thermally pulsing AGB phase and born-again episode. We constrain the stellar mass of PG 0122+200 by comparing the observed period spacing with the asymptotic period spacing and with the average of the computed period spacings. We also employed the individual observed periods to find a representative seismological model for PG 0122+200.

Results. We derive a stellar mass of $0.626 M_{\odot}$ from a comparison between the observed period spacing and the computed asymptotic period spacing, and a stellar mass of $0.567 M_{\odot}$ by comparing the observed period spacing with the average of the computed period spacing. We also find, on the basis of a period-fit procedure, an asteroseismological model representative of PG 0122+200 which is able to reproduce the observed period pattern with an average of the period differences of $\overline{\delta\Pi_i} = 0.88$ s and a root-mean-square residual of $\sigma_{\delta\Pi_i} = 1.27$ s. The model has an effective temperature $T_{\text{eff}} = 81\,500$ K, a stellar mass $M_* = 0.556 M_{\odot}$, a surface gravity $\log g = 7.65$, a stellar luminosity and radius of $\log(L_*/L_{\odot}) = 1.14$ and $\log(R_*/R_{\odot}) = -1.73$, respectively, and a He-rich envelope thickness of $M_{\text{env}} = 1.9 \times 10^{-2} M_{\odot}$. We derive a seismic distance $d \sim 614$ pc and a parallax $\pi \sim 1.6$ mas. The results of the period-fit analysis carried out in this work suggest that the asteroseismological mass of PG 0122+200 could be $\sim 6 - 20\%$ lower than thought hitherto and in closer agreement (to within $\sim 5\%$) with the spectroscopic mass. This result suggests that a reasonable consistency between the stellar mass values obtained from spectroscopy and asteroseismology can be expected when detailed PG1159 evolutionary models are considered.

Key words. stars: evolution — stars: interiors — stars: oscillations — stars: variables: other (GW Virginis)— white dwarfs

1. Introduction

PG 0122+200 (BB Psc or WD 0122+200) is the coolest known pulsating PG1159 star belonging to the GW Vir class of variables. GW Vir stars are very hot hydrogen-deficient post-Asymptotic Giant Branch (AGB) stars with surface layers rich in helium, carbon and oxygen (Werner & Herwig 2006). They exhibit multiperiodic luminosity variations with periods in the range 5–50 minutes, attributable to nonradial pulsation g -modes. PG1159 stars are thought to be the evolutionary link between Wolf-Rayet type central stars of planetary nebulae and most of the hydrogen-deficient white dwarfs. It is generally accepted that these

stars have their origin in a born-again episode induced by a post-AGB helium thermal pulse (see Iben et al. 1983, Herwig et al. 1999, Lawlor & MacDonald 2003, Althaus et al. 2005, Miller Bertolami et al. 2006 for recent references).

PG 0122+200 is characterized by $T_{\text{eff}} = 80\,000 \pm 4\,000$ K and $\log g = 7.5 \pm 0.5$ (Dreizler & Heber 1998). At this effective temperature, PG 0122+200 currently defines the locus of the low-luminosity red edge of the GW Vir instability strip. The photometric variations of this star were discovered by Bond & Grauer (1987). Besides the intrinsic interest in probing its interior, pulsation studies of PG 0122+200 offer a unique opportunity to study neutrino physics. Indeed, at the evolutionary stage characterizing PG 0122+200, neutrino emission constitutes a main energy sink¹ (O’Brien et al. 1998).

Send offprint requests to: A. H. Córscico

* Member of the Carrera del Investigador Científico y Tecnológico, CONICET, Argentina.

** Fellow of CONICET, Argentina.

¹ At variance with the solar neutrino emission, which is a by-product of nuclear fusion, the neutrino flux of pre-white dwarf

The determination of the stellar mass of PG 0122+200 has been the subject of numerous investigations. The stellar mass of pulsating pre-white dwarfs can be constrained, in principle, from asteroseismology—the asteroseismological mass—either through the *observed period spacing* (see, for instance, Kawaler & Bradley 1994; Córscico & Althaus 2006) or by means of the *individual observed periods* (see, e.g., Kawaler & Bradley 1994, Córscico & Althaus 2006, Córscico et al. 2007). The early study of O’Brien et al. (1996) predicts a stellar mass of about $0.66 - 0.72 M_{\odot}$ for PG 0122+200 corresponding to an observed mean period spacing of 21.2 s. Vauclair et al. (1995), on the other hand, suggest an even higher stellar mass, based on a observed mean period spacing of PG 0122+200 of ~ 16 s. Later, O’Brien et al. (1998) find strong evidence for a $\ell = 1$ mean period spacing of 21 s, although a value of ~ 16 s cannot be conclusively ruled out. These values of the period spacing imply a stellar mass of $\sim 0.69 M_{\odot}$ and $\sim 1.0 M_{\odot}$, respectively, based on the PG 1159 models then available. By means of a period-fit procedure based on PG1159 evolutionary models with several masses derived from the full sequence of $0.589 M_{\odot}$ of Althaus et al. (2005), Córscico & Althaus (2006) obtain a stellar mass of $M_* = 0.64 M_{\odot}$ for PG 0122+200. Recently, Fu et al. (2007) (hereinafter FUEA07) have presented new multisite photometric observations of PG 0122+200 obtained in 2001 and 2002. By collecting the new data together with previous observations, these authors have succeeded in detecting a total of 23 frequencies corresponding to modes with $\ell = 1$ and derived unambiguously a mean period spacing of 22.9 s. On the basis of the models of Kawaler & Bradley (1994), these authors inferred a stellar mass of $0.59 \pm 0.02 M_{\odot}$.

The stellar mass of PG1159 stars can also be estimated through the comparison of the spectroscopic values of T_{eff} and g with evolutionary tracks—the spectroscopic mass. On the basis of the evolutionary tracks of O’Brien & Kawaler (2000), Dreizler & Heber (1998) derived a stellar mass of $0.53 \pm 0.1 M_{\odot}$ for PG 0122+200. On the other hand, Werner & Herwig (2006) determined $M_* = 0.58 M_{\odot}$ from a comparison with the H-rich evolutionary models of Schönberner (1983). The most recent determination is that of Miller Bertolami & Althaus (2006), who derived a stellar mass of $0.53 M_{\odot}$ on the basis of PG1159 evolutionary models that take fully into account the evolutionary history and the surface composition of the progenitor stars.

The discrepancy between the asteroseismological mass derived by FUEA07 ($0.57 \leq M_*/M_{\odot} \leq 0.61$) and the most recent spectroscopic determination ($0.53 M_{\odot}$) has prompted us to undertake the present asteroseismological investigation for PG 0122+200, taking full advantage of the new generation of PG1159 evolutionary models recently developed by Miller Bertolami & Althaus (2006). These authors have followed in detail all of the evolutionary phases prior to the formation of PG1159 stars with different stellar masses, particularly the born-again stage. In addition to the issue of the stellar mass, the employment of such detailed PG1159 models allows us to address the question of the He-rich envelope mass ($q_y \equiv M_{\text{env}}/M_{\odot}$) of PG 0122+200, which FUEA07 constrain to be in the range $-6 \lesssim \log q_y \lesssim -5.3$.

stars such as PG 0122+200 is the result of different scattering processes, being the *plasmaneutrino*, *Bremsstrahlung neutrino* and *photoneutrino* emission the most relevant ones (see O’Brien & Kawaler 2000).

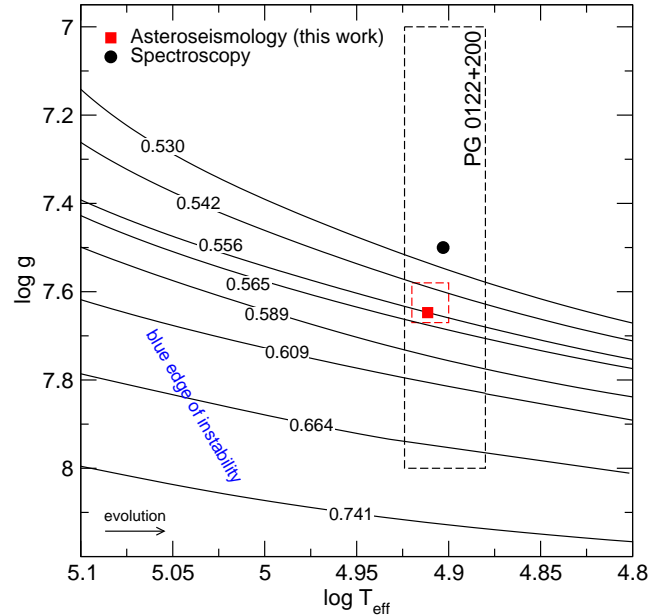


Fig. 1. Our PG1159 full evolutionary tracks in the $\log T_{\text{eff}} - \log g$ plane, labelled with the corresponding stellar mass value in solar units. The black circle is the location of PG 0122+200 according to spectroscopy ($T_{\text{eff}} = 80 \pm 4$ kK and $\log g = 7.5 \pm 0.5$). Note the large error box (dashed), in particular for $\log g$. The square (red) is the location of the star as predicted by our asteroseismological analysis (see §4). The blue (hot) boundary of the theoretical dipole ($\ell = 1$) instability domain—according to Córscico et al. (2006)—is also shown [Color figure only available in the electronic version of the article.]

Finally, a precise knowledge of the mass of PG 0122+200 is a crucial aspect concerning the role played by neutrinos in that star.

The paper is organized as follows: in the next Section we briefly describe our PG1159 evolutionary models. In Sect. 3 we derive the stellar mass of PG 0122+200 by means of the observed period spacing. In Sect. 4 we derive structural parameters of this star by employing the individual observed periods. In this section we derive an asteroseismological model representative of PG 0122+200 (§ 4.1) and discuss its main structural and pulsational characteristics (§ 4.2), its helium envelope thickness (§ 4.3), its mode-trapping properties (§ 4.4) and the asteroseismological distance and parallax (§ 4.5). Finally, in Sect. 5 we summarize our main results and make some concluding remarks.

2. Evolutionary models and numerical tools

The pulsation analysis presented in this work relies on a new generation of stellar models that take into account the complete evolution of PG1159 progenitor stars. These models have been recently employed by our group for a pulsation stability analysis of the GW Vir stars and for an asteroseismological study of the hot PG1159 star RX J2117.1+3412 (Córscico et al. 2006 and Córscico et al. 2007, respectively). The stages for the formation and evolution of PG1159 stars were computed with the LPCODE evolutionary code, which is described at length in Althaus et al. (2005). The neutrino production rates adopted in our computations are those of Itoh et al. (1989, 1992).

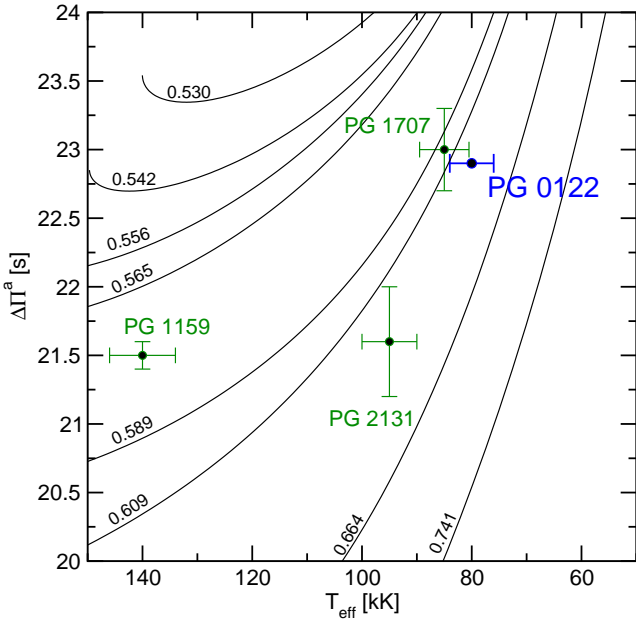


Fig. 2. The dipole ($\ell = 1$) asymptotic period spacing ($\Delta\Pi_\ell^a$) for different stellar masses in terms of the effective temperature. Numbers along each curve denote the stellar masses (in solar units). The plot also shows the location of PG 0122+200 ($T_{\text{eff}} = 80 \pm 4$ kK and $\Delta\Pi^O = 22.9$ s) and the remainder high-gravity, low-luminosity GW Vir stars (PG 1159–035, PG 2131+066, and PG 1707+427) with the period spacing and T_{eff} data taken from Kawaler et al. (2004). The mass of PG 0122+200 as derived by comparing $\Delta\Pi_\ell^a$ with $\Delta\Pi^O$ is $M_* = 0.625^{+0.019}_{-0.016} M_\odot$ [Color figure only available in the electronic version of the article].

Specifically, the background of stellar models was extracted from the evolutionary calculations recently presented by Althaus et al. (2005), Miller Bertolami & Althaus (2006), and Córscico et al. (2006), who computed the complete evolution of model star sequences with initial masses on the ZAMS in the range $1 - 3.75 M_\odot$. We refer the reader to those works for details. Suffice it to mention that all of the post-AGB evolutionary sequences have been followed through the very late thermal pulse (VLTP) and the resulting born-again episode that give rise to the H-deficient, helium-, carbon- and oxygen-rich composition characteristic of PG1159 stars. The masses of the resulting remnants are 0.530, 0.542, 0.556, 0.565, 0.589, 0.609, 0.664, and $0.741 M_\odot$. The new sequence with $M = 0.556 M_\odot$, coming from a progenitor star with $M_* = 1.8 M_\odot$ at the ZAMS, has been computed specifically for the present asteroseismological study. The evolutionary tracks in the $\log T_{\text{eff}} - \log g$ plane for the PG1159 regime are displayed in Fig. 1.

It is worth mentioning that the use of these evolutionary tracks constitutes a major improvement with respect to previous asteroseismological studies. As mentioned, our PG1159 evolutionary sequences are derived from the complete born-again evolution of progenitor stars and a careful treatment of the mixing and extramixing processes during the core helium burning, fundamental aspects when attempts are made at constructing stellar models appropriate for PG1159 stars. In particular, these evolutionary calculations reproduce: (1) the spread in surface chemical composition observed in PG1159 stars, (2) the short born-again times of V4334 Sgr (see Miller Bertolami et al. 2006

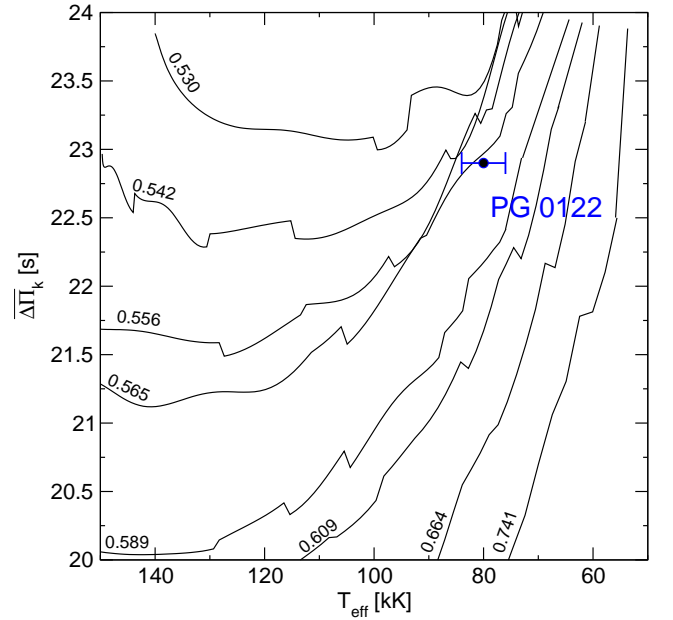


Fig. 3. Same as Fig. 2, but for the average of the computed period spacings ($\Delta\Pi_k$). The mass of PG 0122+200 as derived by comparing $\Delta\Pi_k$ with $\Delta\Pi^O$ is $M_* = 0.567^{+0.007}_{-0.013} M_\odot$ [Color figure only available in the electronic version of the article].

and Miller Bertolami & Althaus 2007a), (3) the location of the GW Vir instability strip in the $\log T_{\text{eff}} - \log g$ plane (Córscico et al. 2006), and (4) the expansion age of the planetary nebula of RX J2117.1+3412 (see the paper by Córscico et al. 2007 and its associated erratum). We believe that the employment of these new evolutionary computations render reliability to our pulsational inferences for PG 0122+200.

We computed $\ell = 1$ g -mode adiabatic pulsation periods and asymptotic period spacings with the same numerical code we employed in our previous works (see, e.g., Córscico & Althaus 2006 for details). We analyzed about 3000 PG1159 models covering a wide range of effective temperatures and luminosities ($5.4 \gtrsim \log(T_{\text{eff}}) \gtrsim 4.8$ and $0 \lesssim \log(L_*/L_\odot) \lesssim 4.2$, respectively) and a range of stellar masses ($0.530 \leq M_*/M_\odot \leq 0.741$).

3. Mass determination from the observed period spacing

Here, we constrain the stellar mass of PG 0122+200 by comparing the asymptotic period spacing, $\Delta\Pi_\ell^a$, and the average of the computed period spacings, $\Delta\Pi_k$ (k being the radial order), with the *observed* period spacing, $\Delta\Pi^O$.² These methods take full advantage of the fact that the period spacing of PG1159 pulsators depends primarily on the stellar mass, and weakly on the luminosity and the He-rich envelope mass fraction (Kawaler & Bradley 1994; Córscico & Althaus 2006). Note that in these approaches we make use of the spectroscopic constraint that the effective temperature of PG 0122+200 is of 80 kK (Dreizler & Heber 1998).

The asymptotic period spacing and the average of the computed period spacings for $\ell = 1$ modes as a func-

² Note that most asteroseismological studies rely on the asymptotic period spacing to infer the stellar mass of GW Vir pulsators.

tion of the effective temperature are displayed in Figs. 2 and 3, respectively, for different stellar masses. Also shown in these diagrams is the location of PG 0122+200, with $\Delta\Pi^O = 22.9$ s (FUEA07). Here, $\Delta\Pi_\ell^a = \Pi_0/\sqrt{\ell(\ell+1)}$, where $\Pi_0 = 2\pi^2[\int_{r_1}^{r_2}(N/r)dr]^{-1}$, being N the Brunt-Väisälä frequency (Tassoul et al. 1990). The quantity $\overline{\Delta\Pi_k}$, on the other hand, is assessed by averaging the computed forward period spacings ($\Delta\Pi_k = \Pi_{k+1} - \Pi_k$) in the range of the observed periods in PG 0122+200 (330–620 s; see Table 1).

From a comparison between $\Delta\Pi^O$ and $\Delta\Pi_\ell^a$ we obtain a stellar mass of $M_* = 0.625_{-0.016}^{+0.019}M_\odot$. The quoted uncertainties in the value of M_* come from the errors in the spectroscopic determination of the effective temperature. In the same way, we get $M_* = 0.567_{-0.013}^{+0.007}M_\odot$ if we compare $\Delta\Pi^O$ and $\overline{\Delta\Pi_k}$. The higher value of M_* (about 10% larger) as derived from $\Delta\Pi_\ell^a$ is due to that usually the asymptotic period spacing is larger than the average of the computed period spacings (see Córscico & Althaus 2006), in particular for the short periods like those exhibited by PG 0122+200, i.e. for which the full asymptotic regime of the modes ($k \gg 1$) has not been attained³. It is important to note that the first method to derive the stellar mass is somewhat less realistic than the second one, because the asymptotic predictions are, in principle, only valid for chemically homogeneous stellar models, while our PG1159 models are indeed chemically stratified.

Finally, we note that our inferred stellar mass values of $M_* \approx 0.57M_\odot$ and in particular $M_* \approx 0.63M_\odot$ are in conflict with the value $M_* = 0.53M_\odot$ as derived from spectroscopy coupled to evolutionary tracks (Dreizler & Heber 1998; Miller Bertolami & Althaus 2006).

4. Constraints from the individual observed periods

4.1. The search for the best-fit model

In this approach we seek a pulsation model that best matches the *individual* pulsation periods of PG 0122+200. We assume that all of the observed periods correspond to $\ell = 1$ modes (see FUEA07). The goodness of the match between the theoretical pulsation periods (Π_k) and the observed individual periods (Π_i^O) is measured by means of a quality function defined as $\chi^2(M_*, T_{\text{eff}}) = \sum_{i=1}^n \min[(\Pi_i^O - \Pi_k)^2]/n$, where n ($= 9$) is the number of observed periods (first column in Table 1). The PG 1159 model that shows the lowest value of χ^2 will be adopted as the “best-fit model”. This approach has also been used by Córscico & Althaus (2006) and Córscico et al. (2007).

We evaluate the function $\chi^2(M_*, T_{\text{eff}})$ for stellar masses of 0.530, 0.542, 0.556, 0.565, 0.589, 0.609, 0.664, and $0.741M_\odot$. For the effective temperature we employed a much more finer grid ($\Delta T_{\text{eff}} = 10 - 30$ K). The quantity $(\chi^2)^{-1}$ in terms of the effective temperature for different stellar masses is shown in the mosaic of Fig. 4 together with the spectroscopic effective temperature of PG 0122+200. We find one strong maximum for the model with $M_* = 0.556M_\odot$ and $T_{\text{eff}} \approx 81.5$ kK (panel c). Such a

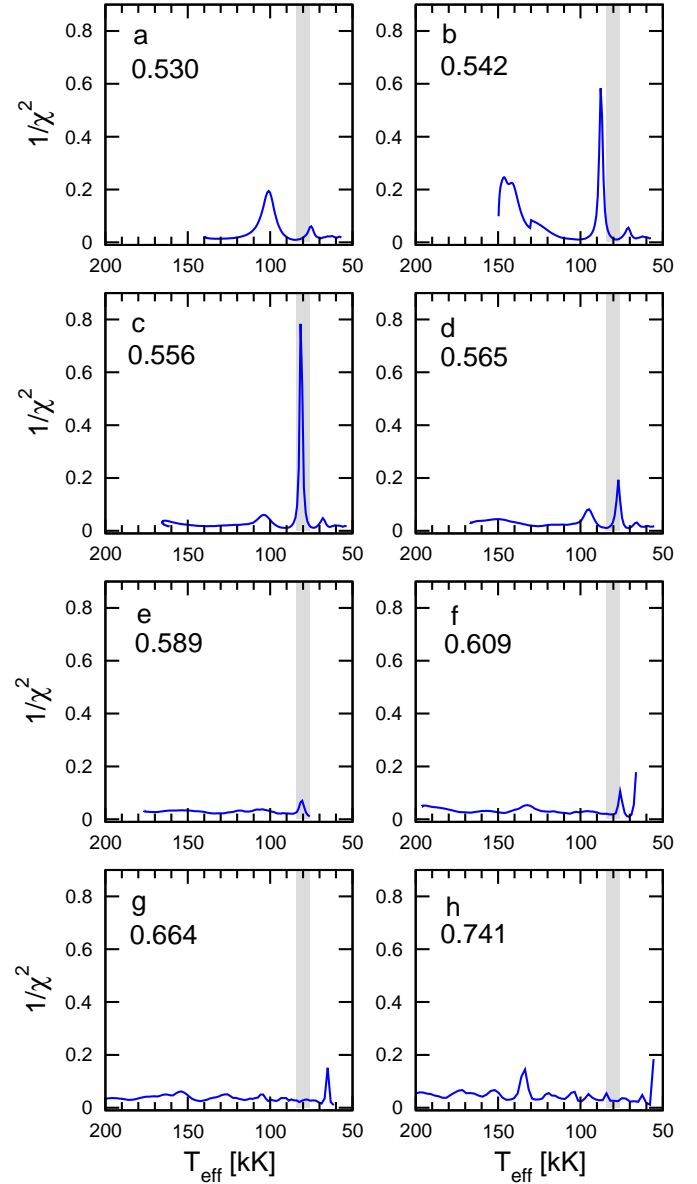


Fig. 4. The inverse of the quality function of the period fit in terms of the effective temperature for the PG1159 sequences with different stellar masses indicated (in solar mass) in each panel. The grey vertical strip corresponds to the spectroscopic effective temperature of PG 0122+200 and its uncertainties ($T_{\text{eff}} = 80\,000 \pm 4\,000$ K). Note the strong maximum in $(\chi^2)^{-1}$ for $M_* = 0.556M_\odot$ at $T_{\text{eff}} \approx 81\,500$ K. This corresponds to our “best-fit” model (see text for details) [Color figure only available in the electronic version of the article].

pronounced maximum in the inverse of χ^2 implies an excellent agreement between the theoretical and observed periods. Another maximum, albeit somewhat less pronounced, is encountered for the model with $M_* = 0.542M_\odot$ at $T_{\text{eff}} \approx 87.7$ kK and constitutes another acceptable asteroseismological solution, in particular because its stellar mass is closer to the spectroscopic mass of PG 0122+200 ($0.53M_\odot$). However, because the agreement between observed and theoretical periods for this model is somewhat poorer than for the one with $M_* = 0.556M_\odot$, we choose to adopt this last model as the best-fit asteroseismological model. Note that our best-fit model has an effective temperature very close to that suggested by spectroscopy,

³ At variance with this, for the longer periods exhibited by RX J2117.1+3412 (with $30 \leq k \leq 53$) the asymptotic conditions are more nearly reached and, as a result, the stellar mass derived from the asymptotic period spacing is very close to that derived from the average of the computed period spacings (see Córscico et al. 2007).

Table 1. Observed $m = 0$ periods (Π_i^O) for PG 0122+200 (taken from FUEA07), theoretical $\ell = 1, m = 0$ periods (Π_k), period differences ($\delta\Pi_i = \Pi_i^O - \Pi_k$), radial orders (k), linear growth rates (η_k), and rates of period change ($\dot{\Pi}_k$) for the best-fit model.

Π_i^O	Π_k	$\delta\Pi_i$	k	η_k	$\dot{\Pi}_k$
[s]	[s]	[s]		[10^{-6}]	[10^{-12} s/s]
336.68	334.12	2.56	12	1.12	1.22
—	354.85	—	13	1.74	1.70
380.10	380.70	-0.60	14	3.54	1.84
400.99	400.35	0.64	15	6.13	1.59
—	425.25	—	16	8.53	2.24
449.48	448.16	1.32	17	15.90	1.71
468.69	469.32	-0.63	18	16.30	2.46
494.92	494.76	0.16	19	28.64	2.19
517.96	516.65	1.31	20	36.50	2.10
—	539.65	—	21	32.24	3.11
564.28	563.98	0.30	22	61.06	2.00
—	585.80	—	23	48.17	3.04
611.15	610.73	0.42	24	55.99	3.26

well inside the error bar. A detailed comparison of the observed $m = 0$ periods in PG 0122+200 with the theoretical periods of the best-fit model is provided in Table 1. The high quality of our period fit is quantitatively reflected by the average of the absolute period differences, $\overline{\delta\Pi_i} = (\sum_{i=1}^n |\delta\Pi_i|)/n$, where $\delta\Pi_i = \Pi_i^O - \Pi_k$, and by the root-mean-square residual, $\sigma_{\delta\Pi_i} = \sqrt{(\sum |\delta\Pi_i|^2)/n}$. We obtain $\overline{\delta\Pi_i} = 0.88$ s and $\sigma_{\delta\Pi_i} = 1.27$ s. The quality of our fit for PG 0122+200 is much better than that achieved by Córscico et al. (2007) for RX J2117.1+3412 ($\overline{\delta\Pi_i} = 1.08$ s) and those obtained by Kawaler & Bradley (1994) and Córscico & Althaus (2006) ($\overline{\delta\Pi_i} = 1.19$ s and $\overline{\delta\Pi_i} = 1.79$ s, respectively) for PG 1159-035. Note that we are able to get a PG1159 model that nicely reproduces the period spectrum observed in PG 0122+200 *without artificially tuning* the value of structural parameters such as the thickness of the outer envelope, the surface chemical abundances, or the shape of the core chemical profile which, instead, are kept fixed at the values predicted by the evolutionary computations.

Table 1 also shows the linear growth rates (η_k) of the fitted pulsation modes (fifth column) for our best-fit model, computed with the nonadiabatic pulsation code described in Córscico et al. (2006). We found that all of the fitted modes have positive values of η_k , implying pulsational instability, although our stability analysis predicts a band of unstable mode-periods ($230 \lesssim \Pi_k \lesssim 730$ s) somewhat wider than the interval of periods detected in PG 0122+200.

The last column in Table 1 shows the rate of period change of the fitted pulsation modes. Our calculations predict all of the pulsation periods to *increase* with time ($\dot{\Pi}_k > 0$), in accordance with the decrease of the Brunt-Väisälä frequency in the core of the model induced by cooling. Note that at the effective temperature of PG 0122+200, cooling has the largest effect on $\dot{\Pi}_k$, while gravitational contraction, which should result in a *decrease* of periods with time, becomes negligible and no longer affects the pulsation periods, except for the case of modes *trapped* in the envelope (see §4.4). Until now, the only secure measurement of $\dot{\Pi}$ in pre-white dwarf stars is that of PG 1159-035, the prototype of the class, for which Costa et al. (1999) obtained a positive

value of $\dot{\Pi} = (+1.307 \pm 0.003) \times 10^{-10}$ s/s for the 516 s period. Note that our theoretical (positive) $\dot{\Pi}_k$ values for the best-fit model ($1.22 - 3.26 \times 10^{-12}$ s/s) are two orders of magnitude lower. For the case of PG 0122+200, a determination of any $\dot{\Pi}$ has not been assessed up to now, although work in this direction is in progress (see Fu et al. 2002).

4.2. Characteristics of the best-fit model

The main features of our best-fit model are summarized in Table 2, where we also include the parameters of PG 0122+200 from other published studies⁴. Note that the effective temperature of our best-fit model is virtually the same as the spectroscopic value. Thus, the location of the star in the $\log T_{\text{eff}} - \log g$ plane is vertically shifted to higher gravities according to our predictions (see Fig. 1).

Our best-fit model has a stellar mass of $M_* = 0.556 M_\odot$, somewhat smaller than the value derived from the average of the computed period spacing, $M_* \sim 0.57 M_\odot$, and substantially lower than that inferred from the asymptotic period spacing, $M_* \sim 0.63 M_\odot$ (see §3). On the other hand, FUEA07 have inferred a value of the stellar mass of PG 0122+200 by using an interpolation formula to the period spacing derived by Kawaler & Bradley (1994) on the basis of a large grid of artificial PG1159 models in the luminosity range $1.6 \lesssim \log(L_*/L_\odot) \lesssim 3.0$. These authors obtain a rather high value of $0.59 \pm 0.02 M_\odot$, in line with the trend of early determinations (O’Brien et al. 1998) and also in good agreement with our values derived from the period spacing, but in clear conflict with the mass of our best-fit model.

On the other hand, the M_* value of our best-fit model is somewhat higher than the spectroscopic mass of $0.53 M_\odot$ derived by Miller Bertolami & Althaus (2006) (see also Dreizler & Heber 1998) for PG 0122+200. Note that a discrepancy between the asteroseismological and the spectroscopic values of M_* is generally encountered among PG1159 pulsators (see Córscico et al. 2006, 2007). Until now, the asteroseismological mass of PG 0122+200 has been about 10 – 30% larger ($\Delta M_* \approx 0.06 - 0.17 M_\odot$) than the spectroscopic mass. In light of the best-fit model derived in this paper, this discrepancy is notably reduced to less than about 5% ($\Delta M_* \approx 0.026 M_\odot$).

FUEA07 infer the stellar luminosity of PG 0122+200 by using the formula of Kawaler & Bradley (1994) mentioned above. They obtain $\log(L_*/L_\odot) = 1.3 \pm 0.5$, larger than the luminosity of our best-fit model, $\log(L_*/L_\odot) = 1.14^{+0.02}_{-0.04}$, and with an accuracy a factor 20 worse. The large uncertainty in the luminosity quoted by FUEA07 is due to the large uncertainty in the spectroscopically determined $\log g$, a quantity used by these authors to derive the luminosity.

4.3. Helium-rich envelope thickness

An important parameter to be discussed separately is the thickness of the outer envelope (M_{env}) of PG 0122+200. We define M_{env} as the mass above the chemical discontinuity between the He-rich envelope and the C/O core.

⁴ Errors in T_{eff} and $\log(L_*/L_\odot)$ are estimated from the width of the maximum in the function χ^2 vs. T_{eff} and $\log(L_*/L_\odot)$, respectively; the error in the stellar mass comes from the grid resolution in M_* . Errors in the remainder quantities are derived from these values.

Table 2. The main characteristics of PG 0122+200. The second column corresponds to spectroscopic results, whereas the third and fourth columns present results from the pulsation study of FUEA07 and from the asteroseismological model of this work, respectively.

Quantity	Spectroscopy	FUEA07	Asteroseismology (This work)
T_{eff} [kK]	$80 \pm 4^{(a)}$	—	$81.54^{+0.8}_{-1.4}$
M_* [M_\odot]	$0.53 \pm 0.1^{(b)}$	0.59 ± 0.02	$0.556^{+0.009}_{-0.014}$
$\log g$ [cm/s ²]	$7.5 \pm 0.5^{(a)}$	—	$7.65^{+0.02}_{-0.07}$
$\log(L_*/L_\odot)$	$1.2^{+0.2}_{-0.3}(**)$	1.3 ± 0.5	$1.14^{+0.02}_{-0.04}$
$\log(R_*/R_\odot)$	$-1.68^{+0.10}_{-0.15}(**)$	-1.65 ± 0.25	$-1.73^{+0.025}_{-0.01}$
M_{env} [M_\odot]	—	$(6 - 30) \times 10^{-7}$	0.019 ± 0.006
C/He, O/He ^(*)	$0.9, 0.4^{(a)}$	—	$1.14, 0.71$
BC [mag]	$-5.81^{+0.23}_{-0.21}$	—	$-5.89^{+0.08}_{-0.04}$
M_V [mag]	$7.55^{+0.74}_{-0.51}$	—	$7.79^{+0.03}_{-0.10}$
M_{bol} [mag]	1.74	—	$1.9^{+0.11}_{-0.14}$
A_V [mag]	0.19	—	0.071
d [pc]	682	700^{+1000}_{-400}	614^{+58}_{-32}
π [mas]	1.47	$1.43^{+1.9}_{-0.84}$	1.6 ± 0.1

Note: (*) Abundances by mass, (**) Interpolated from the tracks by assuming spectroscopic ($T_{\text{eff}}, \log g$) = (80kK, 7.5).
References: (a) Dreizler & Heber (1998); (b) Miller Bertolami & Althaus (2006).

Our best-fit model has $M_{\text{env}} = 0.019M_\odot$. On the other hand, FUEA07 suggest a value of M_{env} in the range $(6 - 30) \times 10^{-7}M_\odot$, about 5 orders of magnitude smaller. In this section, we try to answer the question: could a strikingly low value of M_{env} like that suggested by FUEA07 be explained by mass loss during the PG1159 phase? To this end, we performed additional PG1159 evolutionary calculations to explore the amount of stellar mass that could be eroded by winds. Specifically, we have performed new evolutionary simulations for the sequence of the best-fit model ($M_* = 0.556M_\odot$) starting from the second departure (post-VLTP) of the AGB until the PG1159 stage is reached, with different mass loss rate prescriptions. Specifically, we have adopted two different mass loss rates (\dot{M}_1, \dot{M}_2) appropriate for radiatively driven winds. Namely, the one given by Blöcker (1995), which is based on Pauldrach et al. (1988), results

$$\dot{M}_1 = 1.29 \times 10^{-15} \left(\frac{L_*}{L_\odot} \right)^{1.86} [M_\odot/\text{yr}], \quad (1)$$

and the one adopted by Lawlor & MacDonald (2006), which is based on a modified version of the treatment of Abbott (1982),

$$\dot{M}_2 = 1.2 \times 10^{-15} \left(\frac{L_*}{L_\odot} \right)^2 \left(\frac{M_{\text{eff}}}{M_\odot} \right)^{-1} \left(\frac{Z}{Z_\odot} \right)^{1/2} [M_\odot/\text{yr}]. \quad (2)$$

In the last expression $M_{\text{eff}} = (1 - \Gamma)M_*$ with Γ defined as in Castor et al. (1975). The metallicity was set to $Z = Z_\odot$, because at high metallicities iron lines are expected to be dominant for radiative driven winds (Vink et al. 2001) and iron abundance is expected to remain unchanged during the whole evolution⁵. Roughly, \dot{M}_1 is about one order of magnitude lower than \dot{M}_2 in the present simulations. The total

⁵ However, how important C,N,O lines can be at the extremely high abundances of PG1159 stars is not known. In any case, we think that the inclusion of a simulation with $\dot{M}_3 = 10\dot{M}_2$ —that would correspond to the inclusion of a value $Z/Z_\odot = 100$, or “ $Z = 2$ ” in Eq. (2); see the text—really sets an upper limit for possible mass loss rates during the evolution.

amount of mass lost by these sequences when they reach the location of PG 0122+200 is $7 \times 10^{-5}M_\odot$ for \dot{M}_1 and $4.4 \times 10^{-4}M_\odot$ for \dot{M}_2 , which are both negligible as compared with the mass of the envelope of the best-fit model. For completeness we have considered a more extreme case by adopting a mass loss rate of $\dot{M}_3 = 10\dot{M}_2$. In this case the mass loss rate at the WR-CSPN stage ($L_* \sim 10\,000L_\odot$ and $T_{\text{eff}} < 100\,000$ K) is of the order of several $10^{-6}M_\odot/\text{yr}$, and the rate at the evolutionary “knee” in the HR diagram during the PG1159 stage is of about $10^{-7}M_\odot/\text{yr}$. These values are consistent with the largest rates observed at both PG1159 and WR-CSPN stages (Koesterke et al. 1998, Koesterke 2001) and, consequently, are probably an overestimation of the effect in view of the low mass of our best-fit model. Even in this case, the mass eroded by winds amounts to only $3.4 \times 10^{-3}M_\odot$ which is about one order of magnitude lower than the initial mass of the envelope⁶. Thus, it seems that envelopes as thin as those proposed by FUEA07 could be ruled out in the context of single star stellar evolution. More importantly, the reduction in the mass of the He-rich envelope from a canonical value of $\sim 10^{-2}M_\odot$ to a value of $\sim 10^{-7}M_\odot$ would require an extreme fine tuning (of five orders of magnitude) in the mass-loss rate to avoid the complete removal of the whole envelope. In the absence of a mechanism that justifies this fine tuning, such extremely thin envelopes should be taken with some caution.

4.4. Mode trapping

In this section we shall try to disentangle the possible mode-trapping signatures that could be hidden in the observed period spectrum of PG 0122+200. Following FUEA07, we consider the residuals (R_Π) of the period distribution relative to the mean period spacing⁷. For the

⁶ It is interesting to note that, even for this extreme case, the period-fit does not deteriorate significantly as compared with the case of the best-fit model (see Sect. 4.1).

⁷ Residuals relative to the mean period spacing are more appropriate than the forward period spacing ($\Delta\Pi_k = \Pi_{k+1} - \Pi_k$)

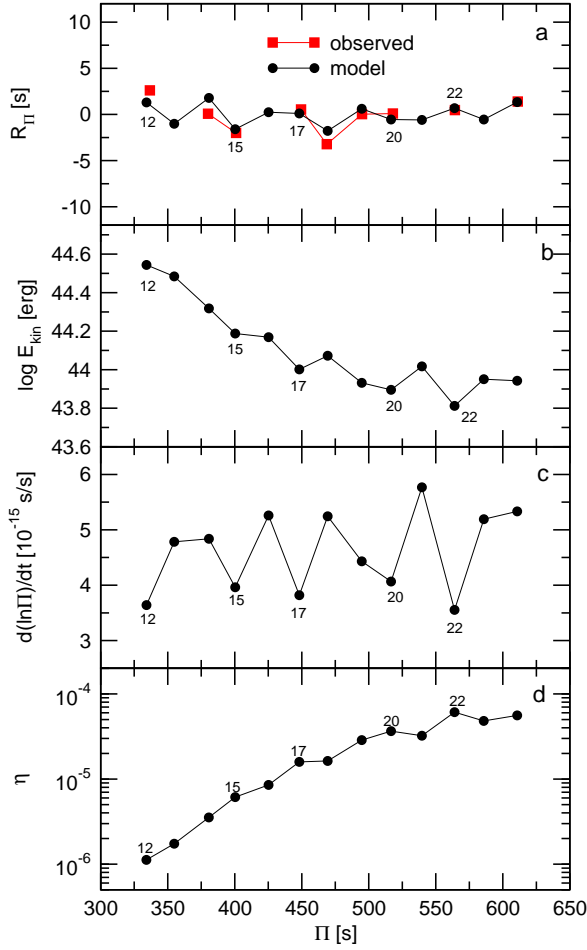


Fig. 5. Panel **a**: Distribution of the residuals R_{Π} relative to the mean period spacing for the case of the observed periods (red) and for the case of the calculated periods (black) of the best-fit model. Panel **b**: the distribution of the kinetic energy. Panel **c**: the values of the relative rates of period change. Panel **d**: the values of the linear nonadiabatic stability coefficients $\eta = -\Im(\sigma)/\Re(\sigma)$ (being σ the complex eigenfrequency). The numbers correspond to the radial order k of the modes trapped in the envelope. See the text for details [Color figure only available in the electronic version of the article].

case of PG 0122+200, a linear least-square fit to the observed periods gives a mean period spacing of 22.97 s, while for the best-fit model we obtain a mean theoretical period spacing of 23.05 s. In panel **a** of Fig. 5 we plot the R_{Π} -distribution for the observed periods (red) and for the case of the calculated periods (black) of the best fit model. The calculated distribution is in very good agreement with the observed one, in particular for the modes with $k = 15, 17, 19, 20, 22, 24$. In addition, the global structure of maxima and minima seen in the observed distribution is nearly duplicated by the computed one.

Mode trapping in PG1159 stars has been discussed at length by Kawaler & Bradley (1994) and Córscico & Althaus (2006); we refer the reader to those works for details. Here, we shall try to answer the question: which modes could be trapped in the outer envelope of PG 0122+200? At first glance, they would be the modes showing a minimum in

the $R_{\Pi} - \Pi$ diagram. As we shall see below, this criterion can lead to erroneous conclusions. A more secure way to find which modes are trapped in the outer envelope is to examine their pulsation kinetic energy (E_{kin}). In panel **b** of Fig. 5 we show the kinetic energy distribution for our best-fit model⁸. Since modes that oscillate mainly in the outer envelope have lower E_{kin} values, one can easily identify trapped modes as those having local minima in the kinetic energy distribution. As can be seen from the figure, they are the modes with $k = 12, 15, 17, 20$ and 22 . Note that in some cases a minimum in R_{Π} does coincide with a minimum in $\log E_{\text{kin}}$ (for instance for $k = 15$) and in other cases does not (for instance for $k = 17$).

Other useful quantities to identify trapped modes are the rates of period changes ($\dot{\Pi}/\Pi$) and the linear stability coefficients (η). Modes trapped in the envelope of the model should “feel” more strongly the effects of the surface gravitational contraction than untrapped modes, and thus the former should be characterized by lower values of $\dot{\Pi}/\Pi$. This is clearly demonstrated in panel **c** of the figure, where we can see that the trapped modes are characterized by local minima in the distribution. On the other hand, it is well known from non-adiabatic arguments that the linear stability coefficients are larger for modes characterized by lower kinetic energies. This is depicted in panel **d** of the figure, where the trapped modes (characterized by low kinetic energies) have local maxima in the η -distribution.

In view of the above discussion, since the mode at $\Pi \approx 468$ s ($k = 18$)—which is identified as a trapped mode by FUEA07—has a minimum in the observed and computed R_{Π} -distributions, but it has a maximum in the kinetic energy, we conclude that this mode is not a trapped mode in the outer envelope. The mode at $\Pi \approx 401$ s ($k = 15$), on the other hand, corresponds to a minimum in the observed and theoretical R_{Π} -distributions, and a minimum of the kinetic energy; so, we conclude that this is a genuine trapped mode in the envelope, confirming the conclusion of FUEA07. However, the trapping cycle of about 68 s ($\Delta k \approx 3$) suggested by FUEA07 is unvalidated in the frame of the present analysis since the mode at $\Pi \approx 468$ s which is used by those authors would not be a trapped mode.

In closing, a final note on the mode-trapping properties of our best-fit model is worth adding. The variations seen in the period distribution—as revealed by the $R_{\Pi} - \Pi$ diagram of Fig. 5—are due to mode-trapping effects inflicted by two chemical transition regions: the inner interface of O/C and the more external interface of O/C/He. The internal chemical profile and the run of the Ledoux term B and the logarithm of the square of the Brunt-Väisälä frequency (N) of our best-fit model in terms of the outer fractional mass are depicted in Fig. 6. We can wonder at this point whether the O/C/He interface or the O/C chemical transition region is more relevant at fixing the mode-trapping pattern of our model, or if there exists a sort of core-envelope degeneracy in the sense that both interfaces are equally effective in producing mode-trapping structure (see Montgomery et al. 2003). To gain some insight into this direction, we have redone our pulsation computations by minimizing the influence of a given chemical interface

—amply used in asteroseismology— because of the many missing modes in the period spectrum of PG 0122+200.

⁸ The kinetic energy values correspond to a normalization of the radial eigenfunction of $\xi_r/r = 1$ at the stellar surface.

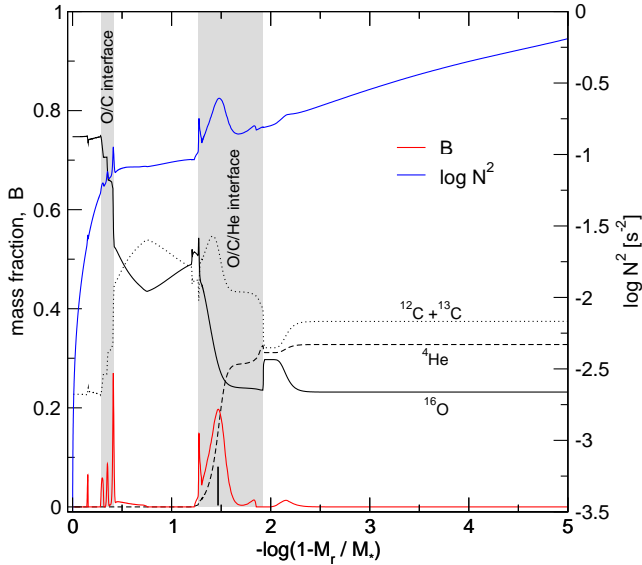


Fig. 6. The internal chemical profile of our best-fit model ($T_{\text{eff}} = 81,540$ K, $M_* = 0.556M_{\odot}$) in terms of the outer fractional mass. Also shown are the profiles of the Ledoux term B and the logarithm of the square of the Brunt-Väisälä frequency (N). The thickness of the outer envelope is $M_{\text{env}} = 0.024M_{\odot}$. The location of the O/C and O/C/He chemical transition regions is emphasized with gray regions. The vertical line at $-\log(1 - M_r/M_*) \sim 1.35$ marks the location of the bottom of the envelope ($X_{\text{He}} \sim 0.14$) [Color figure only available in the electronic version of the article].

on the period structure of the best-fit model⁹. We found that, at the domain of the observed range of periods in PG 0122+200, the mode-trapping features of our model are induced mostly by the chemical gradient at the O/C/He interface, being the O/C interface much less relevant. For periods longer than about 650–700 s, instead, it is the core chemical structure in the O/C interface that mostly fixes the mode trapping properties; this statement applies, for instance, to the cases of PG 1159–035 and RX J2117.1+3412 (see Córscico & Althaus 2005, 2006 and Córscico et al. 2007 for more details).

4.5. The asteroseismological distance and parallax of PG 0122+200

We employ the luminosity of our best-fit model to infer the seismic distance of PG 0122+200 from the Earth. First, we consider the flux predicted by a NLTE model atmosphere with $T_{\text{eff}} = 80$ kK and $\log g = 7.5$ integrated through the spectral response of the V filter. The model atmosphere was calculated with the Tübingen Model Atmosphere Package (see, for details, Werner et al. 2003). We obtain a bolometric correction $BC = -5.89$ and an absolute magnitude $M_V = 7.79$. We account for the interstellar absorption, A_V , using the interstellar extinction model developed by Chen et al. (1998). We compute the seismic distance d according to the well-known relation: $\log d = \frac{1}{5} [m_V - M_V + 5 - A_V(d)]$ where the apparent magnitude is $m_V = 16.8 \pm 0.1$ (FUEA07). The interstellar absorption $A_V(d)$ varies non linearly with the distance and

also depends on the Galactic latitude (b) and longitude (ℓ). For the equatorial coordinates of PG 0122+200 (Epoch B2000.00, $\alpha = 1^{\text{h}} 25^{\text{m}} 22^{\text{s}}.00$, $\delta = +20^{\circ} 17' 54''.0$) the corresponding Galactic coordinates are $b = -41^{\circ} 52' 1''.2$ and $\ell = 133^{\circ} 38' 16''.8$. We solve for d and A_V iteratively and obtain a distance $d = 614_{-32}^{+58}$ pc and an interstellar extinction $A_V = 0.0707$. Note that our distance is $\approx 13\%$ smaller than the estimation of FUEA07 ($d = 700_{-400}^{+1000}$), and with its accuracy substantially improved. Finally, our calculations predict a parallax of $\pi \sim 1.6$ mas.

In closing, we estimate a “spectroscopic” distance of PG 0122+200. We first derive $A_V = 3.1 E(B - V) = 0.19$ by employing the $E(B - V) = 0.06$ value from Dreizler & Heber (1998). The distance can be determined from the model V -flux comparing with m_V and using the spectroscopic T_{eff} and $\log g$ and the extinction A_V . We obtain a spectroscopic distance of 682 pc, and a parallax of ~ 1.47 mas. We derive also an absolute magnitude $M_V = 7.55$ and a bolometric correction $BC = -5.81$ by employing the “spectroscopic” luminosity and radius — interpolated from the tracks assuming spectroscopic T_{eff} and $\log g$ — and the flux predicted by the model atmosphere (see Table 2).

5. Summary and conclusions

In this paper we carried out an asteroseismological study of the cool pulsating PG1159 star PG 0122+200, a g -mode pulsator that defines the red edge of the GW Vir instability domain at low luminosities. Our analysis is based on the full PG1159 evolutionary models of Althaus et al. (2005), Miller Bertolami & Althaus (2006) and Córscico et al. (2006). These models represent a solid basis to analyze the evolutionary and pulsational status of PG1159 stars like PG 0122+200. This is the second GW Vir star that is pulsationally analyzed in the frame of these state-of-the-art PG1159 evolutionary models —the first one being the hottest known GW Vir star, RX J2117.1+3412; see Córscico et al. (2007).

We first took advantage of the strong dependence of the period spacing of variable PG1159 stars on the stellar mass, and derived a value $M_* \sim 0.625M_{\odot}$ by comparing $\Delta\Pi^{\text{O}}$ with the asymptotic period spacing of our models. We also compared $\Delta\Pi^{\text{O}}$ with the computed period spacing averaged over the period range observed in PG 0122+200, and derived a value of $M_* \sim 0.567M_{\odot}$. Note that in both derivations of the stellar mass we made use of the spectroscopic constraint that the effective temperature of the star should be ~ 80 kK. It is interesting to note that the stellar mass as inferred from the asymptotic period spacing is about $0.06M_{\odot}$ larger than that derived from the average of the computed period spacings. This hints at possible systematics in the standard asteroseismological mass determinations methods, in particular when the full asymptotic regime ($k \gg 1$) has not been attained. We note that this systematics in the asteroseismological method is present not only in the case of full PG 1159 evolutionary models as we use here, but also in PG 1159 models artificially created (see Córscico & Althaus 2006). Because most analysis of pulsating PG1159 stars rely on the asymptotic period spacing, this point deserves to be explored for other GW Vir stars, issue which we address in a submitted paper.

Next, we adopted a less conservative approach in which the individual observed pulsation periods alone —i.e., ignoring “external” constraints such as the spectroscopic val-

⁹ We employ the same procedure like in Córscico & Althaus (2005, 2006); we refer the reader to those papers for details.

ues of the surface gravity and effective temperature—naturally lead to an “asteroseismological” PG1159 model that is assumed to be representative of the target star. Specifically, the method consists in looking for the model that best reproduces the observed periods. The period fit was made on a grid of PG1159 models with a quite fine resolution in effective temperature ($\Delta T_{\text{eff}} \sim 10 - 30$ K) although admittedly coarse in stellar mass ($\Delta M_* \sim 0.01 - 0.08 M_\odot$). The match between the periods of the best-fit model and the observed periods in PG 0122+200 turns out to be of an unprecedented quality for this type of studies, being the average of the period differences (observed versus theoretical) of only 0.88 s with a root-mean-square residual of 1.27 s. The stellar mass of the best-fit model is $M_* = 0.556^{+0.009}_{-0.014} M_\odot$.

Interestingly enough, the mass of the best-fit model ($M_* = 0.556^{+0.009}_{-0.014} M_\odot$) is closer to the spectroscopic value of $M_* = 0.53 \pm 0.1 M_\odot$ (Dreizler & Heber 1998; Miller Bertolami & Althaus 2006) than the asteroseismological mass derived in previous works, of $0.59 - 0.69 M_\odot$ (FUEA07; O’Brien et al. 1998).

Other characteristics of the best-fit model are summarized in Table 2. In particular, its effective temperature is nearly the same (to within 2%) as the spectroscopic T_{eff} . The surface gravity, on the other hand, is somewhat larger than the value given by spectroscopy. We also infer the “seismic distance” of PG 0122+200 by using the luminosity of our best-fit model. We obtain a distance $d \sim 614$ pc, somewhat smaller than that of FUEA07.

Finally, our computations predict a temporal period drift for PG 0122+200 between 1.22×10^{-12} s/s and 3.26×10^{-12} s/s. The positive values of $\dot{\Pi}$ (increasing periods) reflect the fact that our best-fit model is entering its white dwarf cooling domain where the effect of the increasing electron degeneracy on the pulsation periods overwhelms that of the surface gravitational contraction, even for the modes trapped in the envelope. Strong theoretical arguments suggest that PG 0122+200 could be used to constrain the plasmon neutrino rates in the dense interior of pre-white dwarfs on the basis of an observed value of $\dot{\Pi}$ (O’Brien et al. 1998; O’Brien & Kawaler 2000). We defer a thorough exploration of this exciting issue to a forthcoming paper.

The results of the period-fit procedure carried out in this work suggest that the asteroseismological mass of PG 0122+200 ($\sim 0.556 M_\odot$) could be $\sim 6 - 20\%$ lower than thought hitherto (see O’Brien et al. 1998 and more recently FUEA07) and in closer agreement (to within $\sim 5\%$) with the spectroscopic mass derived by Miller Bertolami & Althaus (2006). This suggests that a reasonable consistency between the mass values obtained from both (very different) methods should be expected when detailed period-fit procedures on full PG1159 evolutionary models such as those employed in this paper are considered. Even more, *a better agreement between asteroseismological and spectroscopic masses of GW Vir stars could be found when the same evolutionary tracks are used for both the asteroseismological and the spectroscopic derivations of the stellar mass*, as we do in the present work¹⁰. An anomalous case in this context could be RX J2117.1+3412, for which we found an asteroseismological mass about 25% lower than the spectroscopic value by employing the same stellar evolution modeling than here (Córscico et al. 2007). As we suggested in that

paper, the discrepancy in mass could be due to large errors in the spectroscopic determination of $\log g$ and T_{eff} for RX J2117.1+3412, and/or uncertainties in the location of the evolutionary tracks in the HR and $\log T_{\text{eff}} - \log g$ diagrams due to the modeling of PG1159 stars and their precursors¹¹.

In any case, detailed asteroseismological period-fits for other GW Vir stars based on full evolutionary models like we employ here, as well as precise spectroscopic determination of the effective temperature and gravity of PG1159 stars will be needed in the future if we want to reduce the persisting discrepancies in the stellar mass of these fascinating stars.

In closing, in this paper we have been able to find a PG1159 model that nicely reproduces the period spectrum observed in PG 0122+200 *without artificially tuning* the value of structural parameters such as the thickness of the outer envelope, the surface chemical abundances, or the shape of the core chemical profile which, instead, are kept fixed at the values predicted by our evolutionary computations. In some sense, this makes the fit derived more statistically significant. In particular, our PG 1159 evolutionary models are characterized by thick helium-rich envelopes. However, we cannot discard the possibility that pulsating PG1159 star could harbor thin helium-rich envelopes, a possibility sustained by the fact that PG 1159 and born-again stars are observed to suffer from appreciable mass loss. Resulting asteroseismological fits in this case would be worth exploring.

Acknowledgements. This paper has been benefited from a valuable referee report by M. H. Montgomery. This research has been partially supported by the PIP 6521 grant from CONICET. This research has made use of NASA’s Astrophysics Data System.

References

- Abbott, D. C. 1982, ApJ, 259, 282
- Althaus, L. G., Serenelli, A. M., Panei, et al. 2005, A&A, 435, 631
- Blöcker, T. 1995, A&A, 299, 755
- Bond, H., & Grauer, A. 1987, ApJ, 321, L123
- Castor, J. I., Abbott, D. C., & Klein, R. I. 1975, ApJ, 195, 157
- Chen, B., Vergely, J. L., Valette, B., & Carraro, G. 1998, A&A, 336, 137
- Córscico, A. H., & Althaus, L. G. 2005, A&A, 439, L31
- Córscico, A. H., & Althaus, L. G. 2006, A&A, 454, 863
- Córscico, A. H., Althaus, L. G., & Miller Bertolami, M. M. 2006, A&A, 458, 259
- Córscico, A. H., Althaus, L. G., Miller Bertolami, M. M., & Werner, K. 2007, A&A, 461, 1095
- Costa, J. E. S., Kepler, S. O., & Winget, D. E. 1999, ApJ, 522, 973
- Dreizler, S., & Heber, U. 1998, A&A, 334, 618
- Fu, J.-N., Vauclair, G., Solheim, J.-E. et al. 2007, A&A, 467, 237 (FUEA07)
- Fu, J.-N., Vauclair, G., Dolez, N., O’Brien, M. S., & Chevreton, M. (2002), in *Observational Aspects of Pulsating B- & A stars*, eds. C. Sterken & D.W. Kurtz, ASP Conf. Ser., 256, 51
- Herwig, F., Blöcker, T., Langer, N., & Driebe, T. 1999, A&A, 349, L5
- Iben, I., Kaler, J. B., Truran, J. W., & Renzini, A. 1983, ApJ, 264, 605
- Itoh, N., Mutoh, H., Hikita, A., & Kohyama, Y. 1992, ApJ, 395, 622
- Itoh, N., Adachi, T., Nakagawa, M., Kohyama, Y., & Munakata, H. 1989, ApJ, 339, 354
- Kawaler, S. D., et al. 2004, A&A, 428, 969
- Kawaler, S. D., & Bradley, P. A. 1994, ApJ, 427, 415
- Koesterke, L., Dreizler, S., & Rauch, T. 1998, A&A, 330, 1041
- Koesterke, L. 2001, Ap&SS, 275, 41
- Lawlor, T. M., & MacDonald, J. 2003, ApJ, 583, 913

¹⁰ See Quirion et al. (2007) for an enlightening note about this topic.

¹¹ However, recent work by Miller Bertolami & Althaus (2007b) suggests that previous evolution does not play a crucial role in shaping the PG1159 evolutionary tracks.

